

Integration Quiz Review

1. $\int \frac{3x}{9+4x^2} dx$

$$u = 9+4x^2$$

$$\frac{du}{dx} = 8x$$

$$dx = \frac{du}{8x}$$

$$\int \frac{3x}{u} \cdot \frac{du}{8x}$$

$$\frac{3}{8} \int \frac{1}{u} du$$

$$\frac{3}{8} \ln |9+4x^2| + C$$

2. $\int \frac{3x}{\sqrt{-2x^2+5}} dx$

$$u = -2x^2+5 \quad a = \sqrt{5}$$

$$\frac{du}{dx} = -4x$$

$$dx = \frac{du}{-4x}$$

$$\int \frac{3x}{\sqrt{a^2-u}} \cdot \frac{du}{-4x}$$

$$-\frac{3}{4} \int \frac{1}{\sqrt{a^2-u}} du$$

$$-\frac{3}{4} \arcsin \frac{\sqrt{2}x^2}{\sqrt{5}} + C$$

3. $\int 3x^3 \sqrt{4x^2-11} dx$

$$u = 4x^2-11$$

$$\frac{du}{dx} = 8x$$

$$dx = \frac{du}{8x}$$

$$\frac{u+11}{4} = x^2$$

$$\int 3x^3 \cdot u^{1/2} \cdot \frac{du}{8x}$$

$$\frac{3}{8} \int x^2 \cdot u^{1/2} du$$

$$\frac{3}{8} \int \left(\frac{u+11}{4} \right) \cdot u^{1/2} du$$

$$\frac{3}{32} \int u^{3/2} + 11u^{1/2} du$$

$$\frac{3}{32} \left[\frac{2}{5} u^{5/2} + \frac{22}{3} u^{3/2} \right] + C$$

$$\frac{3}{32} \left[\frac{2}{5} (4x^2-11)^{5/2} + \frac{22}{3} (4x^2-11)^{3/2} \right] + C$$

4. $\int \frac{5x^2}{\cos(3x^3)} dx$

$$u = 3x^3$$

$$\frac{du}{dx} = 9x^2$$

$$dx = \frac{du}{9x^2}$$

$$\int \frac{5x^2}{\cos u} \cdot \frac{du}{9x^2}$$

$$\frac{5}{9} \int \sec u du$$

$$\frac{5}{9} \ln |\sec 3x^3 + \tan 3x^3| + C$$

$$5. \int \frac{5x}{x^4+7} dx$$

$$u = x^2 \quad a = \sqrt{7}$$

$$\frac{du}{dx} = 2x$$

$$dx = \frac{du}{2x}$$

$$\int \frac{5x}{a^2+u^2} \cdot \frac{du}{2x}$$

$$\frac{5}{2} \int \frac{1}{a^2+u^2} du$$

$$\frac{5}{2\sqrt{7}} \arctan \frac{x^2}{\sqrt{7}} + C$$

$$6. \int \frac{x^2+5x-3}{x+2} dx \rightarrow \begin{array}{r} x^2+5x-3 \\ x+2 \overline{) 1 \quad 5 \quad -3} \\ \underline{-1 \quad -5 \quad -6} \\ 1 \quad 3 \quad -9 \end{array}$$

$$\int x+3 - \frac{9}{x+2} dx$$

$$\frac{1}{2}x^2 + 3x - 9 \ln|x+2| + C$$

$$7. \int \frac{2x \cot(3x^2)}{\sin(3x^2)} dx$$

$$\int 2x \csc(3x^2) \cot(3x^2) dx$$

$$u = 3x^2$$

$$\frac{du}{dx} = 6x$$

$$dx = \frac{du}{6x}$$

$$\int 2x \csc u \cot u \cdot \frac{du}{6x}$$

$$\frac{1}{3} \int \csc u \cot u du$$

$$-\frac{1}{3} \csc(3x^2) + C$$

$$8. \int \frac{x^2+5x+2}{x^2+3} dx$$

$$\rightarrow \int 1 + \frac{5x-1}{x^2+3} dx$$

$$\begin{array}{r} 1 \\ x^2+3 \overline{) x^2+5x+2} \\ \underline{-x^2 \quad -3} \\ 5x-1 \end{array}$$

$$\int 1 + \frac{5x}{x^2+3} - \frac{1}{x^2+3} dx$$

$$u = x^2+3$$

$$\frac{du}{dx} = 2x$$

$$x + \frac{5}{2} \ln|x^2+3| - \frac{1}{\sqrt{3}} \arctan \frac{x}{\sqrt{3}} + C$$